Real-time Hybrid Simulation (RTHS): Background, Theory and Implementation

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Outline

- Background and theory
- Implementation
  - Issues and challenges
  - NHERI Lehigh solutions
- Example
Background

- Dynamic testing of structures
  - Shake table testing
    - Most realistic method of dynamic testing of structures
    - Limitations:
      - Prototype scaled to accommodate shake table capacity
      - Expensive
  - Hybrid and real-time hybrid simulations (RTHS)
    - Viable alternative to shake table testing
  - Effective force testing
    - Force controlled test and requires all the mass to be present in the lab
    - Limitations:
      - Not economical
      - Force control is more difficult than displacement control
RTHS: Background

- Combines experimental and analytical substructures
  - **Experimental substructure(s)**
    - Not well understood and modeled analytically
    - Full scale component can be easily accommodated
    - Rate dependent devices (e.g., dampers, base-isolators) can be tested
  - **Analytical substructure(s)**
    - Well understood and modeled numerically
    - Various substructures possible for a given expt. substructure
    - Damage can accumulate (not a problem) provided it can be modeled

- Advantages
  - Cost effective large-scale testing method
  - Comprehensive system response
  - Meets the need of the earthquake engineering community
Schematic of RTHS

Structural System

Damping devices

$X_{2,i+1}$

$X_{1,i+1}$

Real-time input ground acceleration

Simulation Coordinator

$M\ddot{X}_{i+1} + C\dot{X}_{i+1} + R^a_{i+1} + R^e_{i+1} = F_{i+1}$

Real-time structural response

Analytical substructure

Experimental substructure (damping devices)
RTHS: Implementation issues and challenges

Analytical substructure

- Fast and accurate state determination procedure for complex structures

Experimental substructure

- Large capacity hydraulic system and dynamic actuators required
- Actuator kinematic compensation
- Robust control of dynamic actuators for large-scale structures

Simulation coordinator

- Numerical integration algorithm
  - Accurate
    - Explicit
    - Unconditionally stable
    - Dissipative
- Fast communication

Preferred
RTHS: Implementation issues and challenges

**Simulation coordinator**

- Numerical integration algorithm
  - Accurate
  - Explicit
  - Unconditionally stable
  - Dissipative

- Fast communication

**NHERI Lehigh Solutions**

- Various explicit model-based algorithms
- RTMD real-time integrated control architecture
Model-based explicit algorithms for RTHS

NHERI Lehigh Solutions to RTHS Challenges

Model-Based Algorithms

Semi-Explicit-\(\alpha\) (SE-\(\alpha\)) Method

Explicit-\(\alpha\) (E-\(\alpha\)) Method

Families of algorithms

Single-parameter families of Algorithms with numerical dissipation

Single-Parameter Semi-Explicit-\(\alpha\) (SSE-\(\alpha\)) Method

Kolay-Ricles-\(\alpha\) (KR-\(\alpha\)) Method

Chen-Ricles (CR) Algorithm

Explicit KR-\(\alpha\) Method

Velocity update: \[ \dot{X}_{i+1} = \dot{X}_i + \Delta t \alpha_1 \ddot{X}_i \]

Displacement update: \[ X_{i+1} = X_i + \Delta t \dot{X}_i + \Delta t^2 \alpha_2 \ddot{X}_i \]

Weighted equations of motion: \[ \mathbf{M} \ddot{X}_{i+1} + \mathbf{C} \dddot{X}_{i+1-\alpha_f} + \mathbf{K} \dot{X}_{i+1-\alpha_f} = \mathbf{F}_{i+1-\alpha_f} \]

where,

\[ \ddot{X}_{i+1} = (I - \alpha_3) \ddot{X}_{i+1} + \alpha_3 \dddot{X}_i \]

\[ \dot{X}_{i+1-\alpha_f} = (1 - \alpha_f) \dot{X}_{i+1} + \alpha_f \dot{X}_i \]

\[ X_{i+1-\alpha_f} = (1 - \alpha_f) X_{i+1} + \alpha_f X_i \]

\[ \mathbf{F}_{i+1-\alpha_f} = (1 - \alpha_f) \mathbf{F}_{i+1} + \alpha_f \mathbf{F}_i \]

Initial acceleration: \[ \mathbf{M} \dddot{X}_0 = [\mathbf{F}_0 - \mathbf{C} \dot{X}_0 - \mathbf{K} X_0] \]

KR-\(\alpha\) Method: Integration Parameters

- **Parameter controlling numerical energy dissipation**
  
  \(\rho_\infty = \text{spectral radius when } \Omega = \omega_n \Delta t \to \infty\)
  - varies in the range \(0 \leq \rho_\infty \leq 1\)
  
  - \(\rho_\infty = 1\): No numerical energy dissipation
    - Algorithm identical to the CR algorithm
  
  - \(\rho_\infty = 0\): Asymptotic annihilation

- **Scalar integration parameters:**
  
  \(\alpha_m = \frac{2}{\rho_\infty - 1} - \frac{1}{\rho_\infty + 1}\)
  \(\alpha_f = \frac{\rho_\infty}{\rho_\infty + 1}\)
  \(\gamma = \frac{1}{2} - \alpha_m + \alpha_f\)
  \(\beta = \frac{1}{4} \left(\frac{1}{2} + \gamma\right)^2\)

- **Matrix integration parameters:**
  
  - \(\alpha_1 = [M + \gamma \Delta t C + \beta \Delta t^2 K]^{-1} M; \quad \alpha_2 = \left(\frac{1}{2} + \gamma\right) \alpha_1\)
  
  - \(\alpha_3 = [M + \gamma \Delta t C + \beta \Delta t^2 K]^{-1} [\alpha_m M + \alpha_f \gamma \Delta t C + \alpha_f \beta \Delta t^2 K]\)

**KR-\(\alpha\): One parameter \((\rho_\infty)\) family of algorithms**
KR-\(\alpha\) Method: Numerical Characteristics

\[ \Omega = \omega_n \Delta t \]

- **Spectral radius**
  - \( \rho_\infty \)
- **Method:** Numerical Characteristics
- **Lower modes of interest** (typ.)
- **Spurious higher modes** (typ.)
- **Equivalent damping ratio**

\[ \xi = 0 \]
\[ \xi = 0.05 \]
KR-α Method: Implementation for RTHS

Initial calculations: specify $\rho_\infty$, calculate $\alpha_f$, $\gamma$, $K^e$, $C^e$, $A$, $B$, $C$, and $D$

Initial conditions: $X_0$, $\dot{X}_0$, $\ddot{X}_0$, and $R_0$

Set $i = 0$

Excitation forces: $F_{i+1-\alpha_f}$
Responses: $X_i$, $\dot{X}_i$, $\ddot{X}_i$, and $R_i$

Optional calculation: $\ddot{X}_{i+1} = D\ddot{X}_{i+1}$

Definitions:
$A = \Delta t \alpha_1 [M - M\alpha_3]^{-1}$
$B = \frac{1}{\Delta t} M\alpha_3 \alpha_1^{-1}$
$D = \frac{1}{\Delta t} \alpha_1^{-1}$
$\ddot{X}_0 = \Delta t \alpha_1 \dddot{X}_0$

Extrapolation Effects – small
($\delta t = \frac{1}{1024}$ s small)

1 Substructure

$D_{i+1}^{c(j)} = X_i^e + \frac{j}{n} (X_{i+1}^e - X_i^e)$

Analytical Substructure

$X_{i+1}^a$ and $\dot{X}_{i+1}^a$

$\ddot{X}_{i+1} = \dddot{X}_i + \ddot{X}_i$

$X_{i+1} = X_i + \Delta t \dddot{X}_i + \left(\frac{1}{2} + \gamma\right)\Delta t \ddot{X}_i$

$R_{i+1}^e = R_{i+1}^{m(n-1)} + K^e \left[ X_{i+1}^e - D_{i+1}^{c(n-1)} \right]$

$+ C^e \left[ \dot{X}_{i+1}^e - V_{i+1}^{c(n-1)} \right]$

$R_{i+1}^a$

$R_{i+1} = R_{i+1}^e + R_{i+1}^a$

$R_{i+1-\alpha_f} = \left(1 - \alpha_f\right)R_{i+1} + \alpha_f R_i$

RTMD Real-time Integrated Control Architecture

NHERI Lehigh Solutions to RTHS Challenges

- Multiple real-time workstations with real-time communication (SCRAMNet)
- Synchronized control commands with simulation data, DAQ, and camera triggers to enable real time simulations and telepresence
RTHS: Implementation issues and challenges

Analytical substructure

- Fast and accurate state determination procedure

NHERI Lehigh Solutions

- HybridFEM
- Multi-grid real-time hybrid simulation
Lehigh HybridFEM
NHERI Lehigh Solutions to RTHS Challenges

- MATLAB and SIMULINK based computational modeling and simulation coordinator software for dynamic time history analysis and real-time hybrid simulation of inelastic-framed structures

- Run Modes
  - MATLAB script for numerical simulation
  - SIMULINK modeling for Real-Time Hybrid simulation with experimental elements via xPCs, and hydraulics-off for training and validation of user algorithms.

- User’s Manual for training

Lehigh HybridFEM

Configuration Options:
• Coordinate system of nodes
• Boundary, constraint and restraint conditions
• Elements
  • Elastic beam-column
  • Elastic spring
  • Inelastic beam-column stress resultant element
  • Non-linear spring
  • Displacement-based NL beam-column fiber element
  • Force-based beam NL column fiber element
  • Zero-length
  • 2D NL planar panel zone
  • Elastic beam-column element with geometric stiffness
• Geometric nonlinearities
• Steel wide flange sections (link to AISC shapes Database)
• Reinforced concrete sections
• Structural mass & inherent damping properties
• Adaptable integration methods

• Materials
  • Elastic
  • Bilinear elasto-plastic
  • Hysteretic
  • Bouc-Wen
  • Trilinear
  • Stiffness degrading
  • Concrete
  • Steel
Multi-grid real-time hybrid simulation

NHERI Lehigh Solutions to RTHS Challenges

- Parallel computing method used with multiple xPCs and SCRAMNet to improve the computational speed for complex large structures

- Incorporated into RTMD Real-time Integrated Control Architecture

RTHS: Implementation issues and challenges

Experimental substructure

- Large capacity hydraulic system and dynamic actuators required
- Actuator kinematic compensation
- Robust control of dynamic actuators for large-scale structures

NHERI Lehigh Solutions

- Large hydraulic power supply system
- 5 large capacity dynamic actuators
- Development of actuator kinematic compensation
- Servo hydraulic actuator control: Adaptive Time Series Compensator (ATS)
Large Capacity Hydraulic System and Dynamic Actuators

NHERI Lehigh Solutions to RTHS Challenges

- Lehigh has unique equipment with large hydraulic power, facilitating large-scale real-time hybrid simulation

**Maximum load capacity**
- 2 actuators: 517 kips (2,300kN)
- 3 actuators: 382 kips (1,700kN)

**Stroke**
- +/- 20 in (+/- 500mm)

**Maximum velocity**
- 45 in/s (1,140mm/sec) for 382 kip actuators
- 33 in/s (840mm/sec) for 517 kip actuators

- Large reaction wall and strong floor

- Accumulator System

- DAQ System

- Large-force capacity dynamic actuators
Large Capacity Hydraulic System and Dynamic Actuators

NRERI Lehigh Solutions to RTHS Challenges

- Lehigh has unique equipment with large hydraulic power, facilitating large-scale real-time hybrid simulation

- Enables a large-scale RTHS of a structure under strong ground motions (i.e., Kobe earthquake, Japan)
- Collapse simulation of a building structure was conducted under extreme earthquake ground motions (beyond MCE level)
Actuator Kinematic Compensation

• Development of kinematic compensation scheme and implementation for RTHS (Mercan et al. 2009)
  – Kinematic correction of command displacements for multi-directional actuator motions
  – Robust, avoiding accumulation of error over multiple time steps; suited for RTHS
  – Exact solution for planar motions

Servo Hydraulic Actuator Control

Sources of Nonlinearity in Real-Time Hybrid Simulation

- Nonlinear servo-valve dynamics
- Nonlinear actuator fluid dynamics
- Test specimen material and geometric nonlinearities
- Slope, misalignment, deformations in test setup

Effect of time delay on real-time hybrid simulation

- Inaccurate structural response
- Delayed restoring force adds energy into the system (negative damping)
- Can cause the instability of simulation

→ Important to negate the time delay effect in real-time hybrid simulation
Servo Hydraulic Actuator Control

- Actuator delay compensation
  - Inverse compensation (Chen 2007)
  - Adaptive inverse compensation (AIC, Chen and Ricles 2010)
  - Adaptive time series (ATS) compensator (Chae et al. 2013)

Adaptive Time Series (ATS) Compensator

2nd order ATS compensator

\[ u^c_k = a_{0k}x^t_k + a_{jk}x^t_k + a_{2k}x^t_k \]

- \( u^c_k \): compensated input displacement into actuator
- \( x^t_k \): target specimen displacement
- \( a_{jk} \): adaptive coefficients

Adaptive coefficients are optimally updated to minimize the error between the specimen target and measured displacements using the least squares method.

\[
A = \left( X_m^T X_m \right)^{-1} X_m^T U_c
\]

- \( A = [a_{0k} a_{1k} \cdots a_{nk}]^T \)
- \( X_m = x^m \dot{x}^m \cdots \frac{d^n}{dt^n}(x^m)^T \)
- \( U_c = u^c_k u^c_{k+1} \cdots u^m_{k+q} \)

Adaptive Time Series (ATS) Compensator
Block Diagram

\[ u_k^c = a_{0k}x_k^t + a_{1k}\dot{x}_k^t + a_{2k}\ddot{x}_k^t \]

Coefficients identification using least squares method

\[ A = \left( X_m^T X_m \right)^{-1} X_m^T U_c \]
Adaptive Time Series (ATS) Compensator

Unique features of ATS compensator

• No user-defined adaptive gains ➞ applicable for large-scale structures susceptible to damage (i.e., concrete structures)

• Negates both variable time delay and variable amplitude error response

• Time delay and amplitude response factor can be easily estimated from the identified values of the coefficients

• Use specimen feedback

\[
\begin{align*}
\text{Amplitude error:} & \quad A = \frac{1}{a_{0k}} \\
\text{Time delay:} & \quad = \frac{a_{1k}}{a_{0k}}
\end{align*}
\]
Adaptive Time Series (ATS) Compensator

- Performance of ATS compensator -

Predefined EQ displacement test (maximum amplitude=40mm)

Slide courtesy of Yunbyeong Chae
Adaptive Time Series (ATS) Compensator

- Performance of ATS compensator -

Predefined EQ displacement test (maximum amplitude=40mm)

\[ \tau = 21 \text{msec} \]

Slide courtesy of Yunbyeong Chae
Adaptive Time Series (ATS) Compensator

- Performance of ATS compensator -

Predefined EQ displacement test (maximum amplitude=40mm)

Slide courtesy of Yunbyeong Chae
Prototype Building

- 3-story, 6-bay by 6-bay office building located in Southern California
- Seismic design category D
- Moment resisting frame (MRF); damped braced frame (DBF), gravity system

Seismic tributary area for one MRF and DBF

Dong, B. “Large-scale Experimental, Numerical, and Design Studies of Steel MRF Structures with Nonlinear Viscous Dampers under Seismic Loading”, PhD Dissertation, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA 2015.
Prototype and Test Structure

- MRFs designed to satisfy ASCE7 code strength requirement
- Story drift controlled by nonlinear elastomeric dampers installed in DBFs
- DBFs designed to remain elastic under design basis earthquake (DBE) ground motion
- Test structures derived by scaling down the prototype by a factor of 0.6

Time discretized weighted equation of motion (KR–α Method):

\[
\ddot{\mathbf{X}}_{i+1} + \mathbf{C}\dot{\mathbf{X}}_{i+1} - \alpha_f + \left(\mathbf{R}^a_{i+1} + \mathbf{R}^e_{i+1} - \alpha_f\right) = \mathbf{F}_{i+1} - \alpha_f
\]

North

East

West

South

Seismic tributary area for one MRF and DBF

X

MRF

Lean-on-col. (gravity system & seismic mass)

Analytical Substructure

Experimental Substructure (DBF)
Inherent and Numerical Damping

- In RTHS using explicit algorithms, generally mass and initial stiffness proportional damping (PD) models are used to model inherent damping in the system:

$$\mathbf{C} = a_0 \mathbf{M} + a_1 \mathbf{K}_I$$

- Known to produce unrealistically large damping forces and inaccurate result when structure undergoes inelastic deformations \((A)\).

- Alternatively nonproportional damping (NPD) can be used:

$$\mathbf{C} = a_0 \mathbf{M} + a_1 \mathbf{K}_I^*$$

- Produces accurate results in nonlinear dynamic analysis using implicit algorithms.

- Produces erroneous results in nonlinear dynamic analysis using explicit algorithms (e.g., CR) with realistic time step size:
  - Member forces become contaminated with participation of spurious higher modes.
  - The problem is worsened by experimental error in RTHS, including the effects of actuator delay compensation algorithms which amplify high frequency signals.

- Numerical damping can be used to circumvent the above problem.

Analytical Substructure

- FE model developed in HybridFEM
- Columns and beams
  - displacement-based nonlinear beam-column fiber elements and elastic beam-column elements
- MRF panel zone
  - nonlinear panel-zone elements
- Nonproportional damping (NPD) model
- Gravity system
  - lean-on-column using elastic elements with second order $P - \Delta$ effects
- 247 DOFs and 74 elements

RTHS: Ground motion and time step

- **Ground motion**
  - B-WSM180 component of the 1987 Superstition Hills earthquake recorded at the Westmoreland Fire Station
  - Chosen from a suit of 20 ground motion records which produce a median spectral acceleration that matches the design spectrum in the period range of 0.2 – 2.0 sec.
  - Scaled to two hazard levels
    - Design basis earthquake (DBE)*: Scale factor = 1.51
    - Maximum considered earthquake (MCE)*: Scale factor = 2.26

- **Time step**
  - $\Delta = \frac{4}{1024}$ sec, the smallest time step within which the numerical computation can be finished in real-time

*Note: DBE has 475 year return period (10% probability of exceedance in 50 years)
MCE has 2475 year return period (2% probability of exceedance in 50 years)
MCE level RTHS using $\rho_\infty = 1.0$

Freq. $\approx f_{Nqy} = \frac{1}{2\Delta t}$

High frequency oscillations in member forces

- Under nonlinear structural behavior, pulses are introduced in the acceleration at the Nyquist frequency \( \left( = \frac{1}{2\Delta t} \right) \) when the state of the structure changes within the time step.

- These pulses excite spurious higher modes present in the system which primarily contribute to the member forces.

- The problem becomes worst by the noise introduced through the measured restoring forces and the actuator delay compensation which can amplify high frequency noise.

- How can we remove them?
  - Reduce the time step: Not always possible due to the computation time required for each time step.
  - Introduce controllable numerical damping.
MCE level RTHS using $\rho_\infty = 0.75$

**Actuator control: Typical MCE level test & $\rho_\infty = 0.75$**

<table>
<thead>
<tr>
<th>Error indices</th>
<th>Floor-1</th>
<th>Floor-2</th>
<th>Floor-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. amp. error (%)</td>
<td>0.27</td>
<td>0.46</td>
<td>0.91</td>
</tr>
<tr>
<td>NEE (%)</td>
<td>0.04</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>NRMSE (%)</td>
<td>0.29</td>
<td>0.14</td>
<td>0.13</td>
</tr>
</tbody>
</table>

\[
A^{(j)}_k \approx \frac{1}{a^{(j)}_{0k}} \\
\tau^{(j)}_k \approx \frac{a^{(j)}_{1k}}{a^{(j)}_{0k}}
\]

Max. amp. error = \[
\frac{\max |x^t| - \max |x^m|}{\max |x^t|}
\]

NEE = \[
\frac{\sum_{i=1}^{n} x^t_i - \sum_{i=1}^{n} x^m_i}{\sum_{i=1}^{n} x^m_i} \]

NRMSE = \[
\frac{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (x^t_i - x^m_i)^2}}{\max(x^m) - \min(x^m)}\]

$x^t$: targeted specimen displacement  
$u^c$: input command to controller  
$x^m$: measured specimen displacement
Summary

- Reviewed the concept of RTHS
- NHERI Lehigh Capabilities for conducting RTHS
  - RTMD integrated control architecture
  - Various model-based explicit unconditionally stable algorithms with controllable numerical dissipation
  - Nonlinear computational modeling program: HybridFEM
  - Multigrid hybrid simulation capabilities
  - Large capacity hydraulic systems and dynamic actuators
  - Advanced actuator control: Adaptive Time Series (ATS) Compensator
- Future developments
  - Hybrid simulations for wind loading
  - Hybrid simulations including soil-structure interaction
Thank you